4.1 Exercises

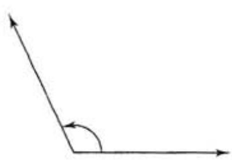
The *HM mathSpace®* CD-ROM and *Eduspace®* for this text contain step-by-step solutions to all odd-numbered exercises. They also provide Tutorial Exercises for additional help.

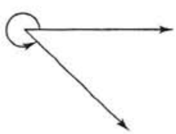
VOCABULARY CHECK: Fill in the blanks.

1. ________ means "measurement of triangles."
2. An ________ is determined by rotating a ray about its endpoint.
3. Two angles that have the same initial and terminal sides are ________.
4. One ________ is the measure of a central angle that intercepts an arc equal to the radius of the circle.
5. Angles that measure between 0 and $\pi/2$ are ________ angles, and angles that measure between $\pi/2$ and π are ________ angles.
6. Two positive angles that have a sum of $\pi/2$ are ________ angles, whereas two positive angles that have a sum of π are ________ angles.
7. The angle measure that is equivalent to $\frac{1}{360}$ of a complete revolution about an angle's vertex is one ________.
8. The ________ speed of a particle is the ratio of the arc length traveled to the time traveled.
9. The ________ speed of a particle is the ratio of the change in the central angle to time.
10. The area of a sector of a circle with radius r and central angle θ, where θ is measured in radians, is given by the formula ________.

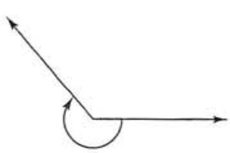
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

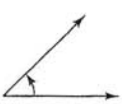
In Exercises 1–6, estimate the angle to the nearest one-half radian.

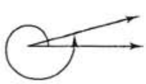
1. 

2. 

3. 

4. 

5. 

6. 

In Exercises 7–12, determine the quadrant in which each angle lies. (The angle measure is given in radians.)

7. (a) $\frac{\pi}{5}$ (b) $\frac{7\pi}{5}$

8. (a) $\frac{11\pi}{8}$ (b) $\frac{9\pi}{8}$

9. (a) $-\frac{\pi}{12}$ (b) -2

10. (a) -1 (b) $-\frac{11\pi}{9}$

11. (a) 3.5 (b) 2.25

12. (a) 6.02 (b) -4.25

In Exercises 13–16, sketch each angle in standard position.

13. (a) $\frac{5\pi}{4}$ (b) $-\frac{2\pi}{3}$

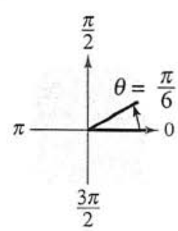
14. (a) $-\frac{7\pi}{4}$ (b) $\frac{5\pi}{2}$

15. (a) $\frac{11\pi}{6}$ (b) -3

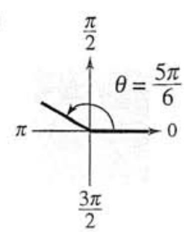
16. (a) 4 (b) 7π

In Exercises 17–20, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in radians.

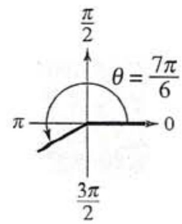
17. (a)



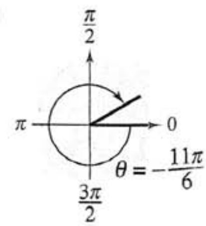
(b)



18. (a)



(b)



19. (a) $\theta = \frac{2\pi}{3}$ (b) $\theta = \frac{\pi}{12}$

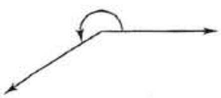
20. (a) $\theta = -\frac{9\pi}{4}$ (b) $\theta = -\frac{2\pi}{15}$

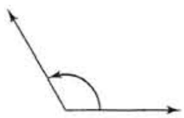
In Exercises 21–24, find (if possible) the complement and supplement of each angle.

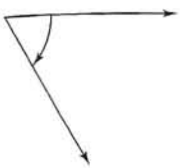
21. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ **22.** (a) $\frac{\pi}{12}$ (b) $\frac{11\pi}{12}$

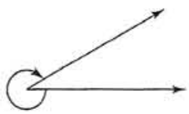
23. (a) 1 (b) 2 **24.** (a) 3 (b) 1.5

In Exercises 25–30, estimate the number of degrees in the angle.

25.


26.


27.


28.


29.


30.


In Exercises 31–34, determine the quadrant in which each angle lies.

31. (a) 130° (b) 285°

32. (a) 8.3° (b) $257^\circ 30'$

33. (a) $-132^\circ 50'$ (b) -336°

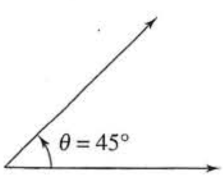
34. (a) -260° (b) -3.4°

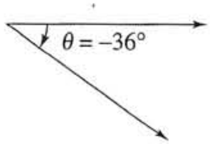
In Exercises 35–38, sketch each angle in standard position.

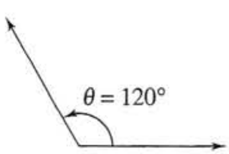
35. (a) 30° (b) 150° **36.** (a) -270° (b) -120°

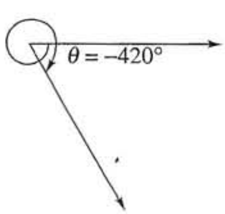
37. (a) 405° (b) 480° **38.** (a) -750° (b) -600°

In Exercises 39–42, determine two coterminal angles (one positive and one negative) for each angle. Give your answers in degrees.

39. (a)


(b)


40. (a)


(b)


41. (a) $\theta = 240^\circ$ (b) $\theta = -180^\circ$

42. (a) $\theta = -420^\circ$ (b) $\theta = 230^\circ$

In Exercises 43–46, find (if possible) the complement and supplement of each angle.

43. (a) 18° (b) 115° **44.** (a) 3° (b) 64°

45. (a) 79° (b) 150° **46.** (a) 130° (b) 170°

In Exercises 47–50, rewrite each angle in radian measure as a multiple of π. (Do not use a calculator.)

47. (a) 30° (b) 150° **48.** (a) 315° (b) 120°

49. (a) -20° (b) -240° **50.** (a) -270° (b) 144°

In Exercises 51–54, rewrite each angle in degree measure. (Do not use a calculator.)

51. (a) $\frac{3\pi}{2}$ (b) $\frac{7\pi}{6}$ **52.** (a) $-\frac{7\pi}{12}$ (b) $\frac{\pi}{9}$

53. (a) $\frac{7\pi}{3}$ (b) $-\frac{11\pi}{30}$ **54.** (a) $\frac{11\pi}{6}$ (b) $\frac{34\pi}{15}$

In Exercises 55–62, convert the angle measure from degrees to radians. Round to three decimal places.

55. 115° **56.** 87.4°

57. -216.35° **58.** -48.27°

59. 532° **60.** 345°

61. -0.83° **62.** 0.54°

In Exercises 63–70, convert the angle measure from radians to degrees. Round to three decimal places.

63. $\frac{\pi}{7}$ **64.** $\frac{5\pi}{11}$

65. $\frac{15\pi}{8}$ **66.** $\frac{13\pi}{2}$

67. -4.2π **68.** 4.8π

69. -2 **70.** -0.57

In Exercises 71–74, convert each angle measure to decimal degree form.

71. (a) $54^\circ 45'$ (b) $-128^\circ 30'$

72. (a) $245^\circ 10'$ (b) $2^\circ 12'$

73. (a) $85^\circ 18' 30''$ (b) $330^\circ 25''$

74. (a) $-135^\circ 36''$ (b) $-408^\circ 16' 20''$

In Exercises 75–78, convert each angle measure to D° M′ S″ form.

75. (a) 240.6° (b) -145.8°

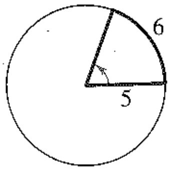
76. (a) -345.12° (b) 0.45°

77. (a) 2.5° (b) -3.58°

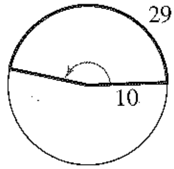
78. (a) -0.355° (b) 0.7865°

In Exercises 79–82, find the angle in radians.

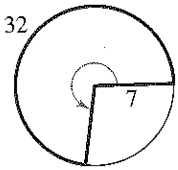
79.



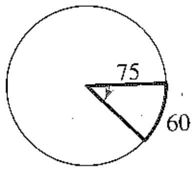
80.



81.



82.



In Exercises 83–86, find the radian measure of the central angle of a circle of radius r that intercepts an arc of length s.

	Radius r	*Arc Length s*
83.	27 inches	6 inches
84.	14 feet	8 feet
85.	14.5 centimeters	25 centimeters
86.	80 kilometers	160 kilometers

In Exercises 87–90, find the length of the arc on a circle of radius r intercepted by a central angle θ.

	Radius r	*Central Angle θ*
87.	15 inches	$180°$
88.	9 feet	$60°$
89.	3 meters	1 radian
90.	20 centimeters	$\pi/4$ radian

In Exercises 91–94, find the area of the sector of the circle with radius r and central angle θ.

	Radius r	*Central Angle θ*
91.	4 inches	$\dfrac{\pi}{3}$
92.	12 millimeters	$\dfrac{\pi}{4}$
93.	2.5 feet	$225°$
94.	1.4 miles	$330°$

Distance Between Cities **In Exercises 95 and 96, find the distance between the cities. Assume that Earth is a sphere of radius 4000 miles and that the cities are on the same longitude (one city is due north of the other).**

	City	*Latitude*
95.	Dallas, Texas	32° 47′ 39″ N
	Omaha, Nebraska	41° 15′ 50″ N
96.	San Francisco, California	37° 47′ 36″ N
	Seattle, Washington	47° 37′ 18″ N

97. ***Difference in Latitudes*** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Syracuse, New York and Annapolis, Maryland, where Syracuse is 450 kilometers due north of Annapolis?

98. ***Difference in Latitudes*** Assuming that Earth is a sphere of radius 6378 kilometers, what is the difference in the latitudes of Lynchburg, Virginia and Myrtle Beach, South Carolina, where Lynchburg is 400 kilometers due north of Myrtle Beach?

99. ***Instrumentation*** The pointer on a voltmeter is 6 centimeters in length (see figure). Find the angle through which the pointer rotates when it moves 2.5 centimeters on the scale.

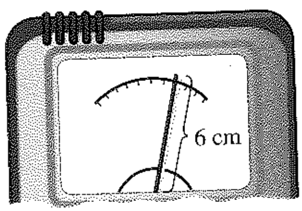


FIGURE FOR 99

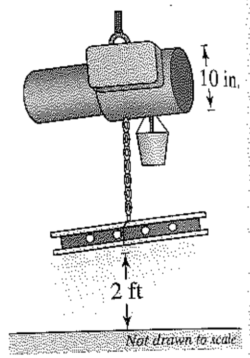


FIGURE FOR 100

100. ***Electric Hoist*** An electric hoist is being used to lift a beam (see figure). The diameter of the drum on the hoist is 10 inches, and the beam must be raised 2 feet. Find the number of degrees through which the drum must rotate.

101. ***Angular Speed*** A car is moving at a rate of 65 miles per hour, and the diameter of its wheels is 2.5 feet.

(a) Find the number of revolutions per minute the wheels are rotating.

(b) Find the angular speed of the wheels in radians per minute.

102. ***Angular Speed*** A two-inch-diameter pulley on an electric motor that runs at 1700 revolutions per minute is connected by a belt to a four-inch-diameter pulley on a saw arbor.

(a) Find the angular speed (in radians per minute) of each pulley.

(b) Find the revolutions per minute of the saw.

103. ***Linear and Angular Speeds*** A $7\frac{1}{4}$-inch circular power saw rotates at 5200 revolutions per minute.

(a) Find the angular speed of the saw blade in radians per minute.

(b) Find the linear speed (in feet per minute) of one of the 24 cutting teeth as they contact the wood being cut.

104. ***Linear and Angular Speeds*** A carousel with a 50-foot diameter makes 4 revolutions per minute.

(a) Find the angular speed of the carousel in radians per minute.

(b) Find the linear speed of the platform rim of the carousel.

105. ***Linear and Angular Speeds*** The diameter of a DVD is approximately 12 centimeters. The drive motor of the DVD player is controlled to rotate precisely between 200 and 500 revolutions per minute, depending on what track is being read.

(a) Find an interval for the angular speed of a DVD as it rotates.

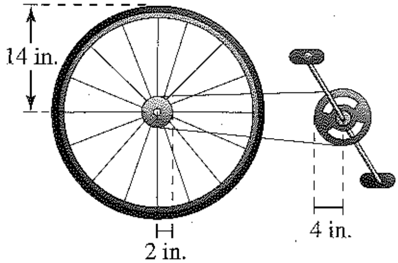
(b) Find an interval for the linear speed of a point on the outermost track as the DVD rotates.

106. ***Area*** A car's rear windshield wiper rotates 125°. The total length of the wiper mechanism is 25 inches and wipes the windshield over a distance of 14 inches. Find the area covered by the wiper.

107. ***Area*** A sprinkler system on a farm is set to spray water over a distance of 35 meters and to rotate through an angle of 140°. Draw a diagram that shows the region that can be irrigated with the sprinkler. Find the area of the region.

Model It

108. ***Speed of a Bicycle*** The radii of the pedal sprocket, the wheel sprocket, and the wheel of the bicycle in the figure are 4 inches, 2 inches, and 14 inches, respectively. A cyclist is pedaling at a rate of 1 revolution per second.



(a) Find the speed of the bicycle in feet per second and miles per hour.

(b) Use your result from part (a) to write a function for the distance d (in miles) a cyclist travels in terms of the number n of revolutions of the pedal sprocket.

Model It (continued)

(c) Write a function for the distance d (in miles) a cyclist travels in terms of the time t (in seconds). Compare this function with the function from part (b).

(d) Classify the types of functions you found in parts (b) and (c). Explain your reasoning.

Synthesis

True or False? **In Exercises 109–111, determine whether the statement is true or false. Justify your answer.**

109. A measurement of 4 radians corresponds to two complete revolutions from the initial side to the terminal side of an angle.

110. The difference between the measures of two coterminal angles is always a multiple of 360° if expressed in degrees and is always a multiple of 2π radians if expressed in radians.

111. An angle that measures $-1260°$ lies in Quadrant III.

112. ***Writing*** In your own words, explain the meanings of (a) an angle in standard position, (b) a negative angle, (c) coterminal angles, and (d) an obtuse angle.

113. ***Think About It*** A fan motor turns at a given angular speed. How does the speed of the tips of the blades change if a fan of greater diameter is installed on the motor? Explain.

114. ***Think About It*** Is a degree or a radian the larger unit of measure? Explain.

115. ***Writing*** If the radius of a circle is increasing and the magnitude of a central angle is held constant, how is the length of the intercepted arc changing? Explain your reasoning.

116. ***Proof*** Prove that the area of a circular sector of radius r with central angle θ is $A = \frac{1}{2}\theta r^2$, where θ is measured in radians.

Skills Review

In Exercises 117–120, simplify the radical expression.

117. $\dfrac{4}{4\sqrt{2}}$ 118. $\dfrac{5\sqrt{5}}{2\sqrt{10}}$

119. $\sqrt{2^2 + 6^2}$ 120. $\sqrt{17^2 - 9^2}$

In Exercises 121–124, sketch the graphs of $y = x^5$ and the specified transformation.

121. $f(x) = (x - 2)^5$ 122. $f(x) = x^5 - 4$

123. $f(x) = 2 - x^5$ 124. $f(x) = -(x + 3)^5$

4.3 Exercises

VOCABULARY CHECK:

1. Match the trigonometric function with its right triangle definition.

(a) Sine (b) Cosine (c) Tangent (d) Cosecant (e) Secant (f) Cotangent

(i) $\dfrac{\text{hypotenuse}}{\text{adjacent}}$ (ii) $\dfrac{\text{adjacent}}{\text{opposite}}$ (iii) $\dfrac{\text{hypotenuse}}{\text{opposite}}$ (iv) $\dfrac{\text{adjacent}}{\text{hypotenuse}}$ (v) $\dfrac{\text{opposite}}{\text{hypotenuse}}$ (vi) $\dfrac{\text{opposite}}{\text{adjacent}}$

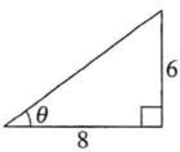
In Exercises 2 and 3, fill in the blanks.

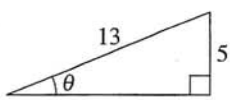
2. Relative to the angle θ, the three sides of a right triangle are the ________ side, the ________ side, and the ________.

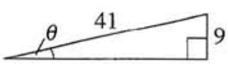
3. An angle that measures from the horizontal upward to an object is called the angle of ________, whereas an angle that measures from the horizontal downward to an object is called the angle of ________.

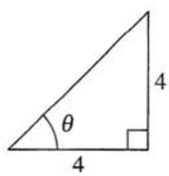
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, find the exact values of the six trigonometric functions of the angle θ shown in the figure. (Use the Pythagorean Theorem to find the third side of the triangle.)

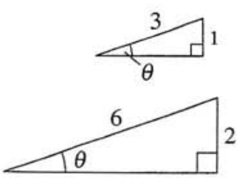
1.


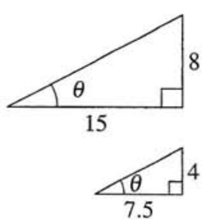
2.


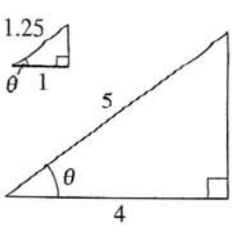
3.


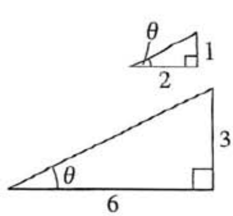
4.


In Exercises 5–8, find the exact values of the six trigonometric functions of the angle θ for each of the two triangles. Explain why the function values are the same.

5.


6.


7.


8.


In Exercises 9–16, sketch a right triangle corresponding to the trigonometric function of the acute angle θ. Use the Pythagorean Theorem to determine the third side and then find the other five trigonometric functions of θ.

9. $\sin\theta = \frac{3}{4}$

10. $\cos\theta = \frac{5}{7}$

11. $\sec\theta = 2$

12. $\cot\theta = 5$

13. $\tan\theta = 3$

14. $\sec\theta = 6$

15. $\cot\theta = \frac{3}{2}$

16. $\csc\theta = \frac{17}{4}$

In Exercises 17–26, construct an appropriate triangle to complete the table. ($0 \le \theta \le 90°$, $0 \le \theta \le \pi/2$)

	Function	*θ (deg)*	*θ (rad)*	*Function Value*
17.	sin	30°		
18.	cos	45°		
19.	tan		$\frac{\pi}{3}$	
20.	sec		$\frac{\pi}{4}$	
21.	cot			$\frac{\sqrt{3}}{3}$
22.	csc			$\sqrt{2}$
23.	cos		$\frac{\pi}{6}$	
24.	sin		$\frac{\pi}{4}$	
25.	cot			1
26.	tan			$\frac{\sqrt{3}}{3}$

In Exercises 27–32, use the given function value(s), and trigonometric identities (including the cofunction identities), to find the indicated trigonometric functions.

27. $\sin 60^\circ = \frac{\sqrt{3}}{2}$, $\cos 60^\circ = \frac{1}{2}$

(a) $\tan 60^\circ$ (b) $\sin 30^\circ$

(c) $\cos 30^\circ$ (d) $\cot 60^\circ$

28. $\sin 30^\circ = \frac{1}{2}$, $\tan 30^\circ = \frac{\sqrt{3}}{3}$

(a) $\csc 30^\circ$ (b) $\cot 60^\circ$

(c) $\cos 30^\circ$ (d) $\cot 30^\circ$

29. $\csc \theta = \frac{\sqrt{13}}{2}$, $\sec \theta = \frac{\sqrt{13}}{3}$

(a) $\sin \theta$ (b) $\cos \theta$

(c) $\tan \theta$ (d) $\sec(90^\circ - \theta)$

30. $\sec \theta = 5$, $\tan \theta = 2\sqrt{6}$

(a) $\cos \theta$ (b) $\cot \theta$

(c) $\cot(90^\circ - \theta)$ (d) $\sin \theta$

31. $\cos \alpha = \frac{1}{3}$

(a) $\sec \alpha$ (b) $\sin \alpha$

(c) $\cot \alpha$ (d) $\sin(90^\circ - \alpha)$

32. $\tan \beta = 5$

(a) $\cot \beta$ (b) $\cos \beta$

(c) $\tan(90^\circ - \beta)$ (d) $\csc \beta$

In Exercises 33–42, use trigonometric identities to transform the left side of the equation into the right side $(0 < \theta < \pi/2)$.

33. $\tan \theta \cot \theta = 1$

34. $\cos \theta \sec \theta = 1$

35. $\tan \alpha \cos \alpha = \sin \alpha$

36. $\cot \alpha \sin \alpha = \cos \alpha$

37. $(1 + \cos \theta)(1 - \cos \theta) = \sin^2 \theta$

38. $(1 + \sin \theta)(1 - \sin \theta) = \cos^2 \theta$

39. $(\sec \theta + \tan \theta)(\sec \theta - \tan \theta) = 1$

40. $\sin^2 \theta - \cos^2 \theta = 2 \sin^2 \theta - 1$

41. $\frac{\sin \theta}{\cos \theta} + \frac{\cos \theta}{\sin \theta} = \csc \theta \sec \theta$

42. $\frac{\tan \beta + \cot \beta}{\tan \beta} = \csc^2 \beta$

In Exercises 43–52, use a calculator to evaluate each function. Round your answers to four decimal places. (Be sure the calculator is in the correct angle mode.)

43. (a) $\sin 10^\circ$ (b) $\cos 80^\circ$

44. (a) $\tan 23.5^\circ$ (b) $\cot 66.5^\circ$

45. (a) $\sin 16.35^\circ$ (b) $\csc 16.35^\circ$

46. (a) $\cos 16^\circ 18'$ (b) $\sin 73^\circ 56'$

47. (a) $\sec 42^\circ 12'$ (b) $\csc 48^\circ 7'$

48. (a) $\cos 4^\circ 50' 15''$ (b) $\sec 4^\circ 50' 15''$

49. (a) $\cot 11^\circ 15'$ (b) $\tan 11^\circ 15'$

50. (a) $\sec 56^\circ 8\ 10''$ (b) $\cos 56^\circ 8\ 10''$

51. (a) $\csc 32^\circ 40' 3''$ (b) $\tan 44^\circ 28\ 16''$

52. (a) $\sec\left(\frac{9}{5} \cdot 20 + 32\right)^\circ$ (b) $\cot\left(\frac{9}{5} \cdot 30 + 32\right)^\circ$

In Exercises 53–58, find the values of θ in degrees $(0^\circ < \theta < 90^\circ)$ and radians $(0 < \theta < \pi/2)$ without the aid of a calculator.

53. (a) $\sin \theta = \frac{1}{2}$ (b) $\csc \theta = 2$

54. (a) $\cos \theta = \frac{\sqrt{2}}{2}$ (b) $\tan \theta = 1$

55. (a) $\sec \theta = 2$ (b) $\cot \theta = 1$

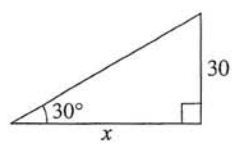
56. (a) $\tan \theta = \sqrt{3}$ (b) $\cos \theta = \frac{1}{2}$

57. (a) $\csc \theta = \frac{2\sqrt{3}}{3}$ (b) $\sin \theta = \frac{\sqrt{2}}{2}$

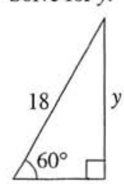
58. (a) $\cot \theta = \frac{\sqrt{3}}{3}$ (b) $\sec \theta = \sqrt{2}$

In Exercises 59–62, solve for *x*, *y*, or *r* as indicated.

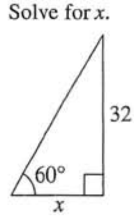
59. Solve for x.



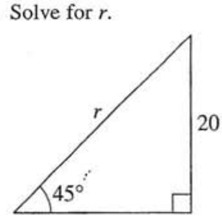
60. Solve for y.



61. Solve for x.



62. Solve for r.



63. ***Empire State Building*** You are standing 45 meters from the base of the Empire State Building. You estimate that the angle of elevation to the top of the 86th floor (the observatory) is 82°. If the total height of the building is another 123 meters above the 86th floor, what is the approximate height of the building? One of your friends is on the 86th floor. What is the distance between you and your friend?

4.4 Exercises

VOCABULARY CHECK:

In Exercises 1–6, let θ be an angle in standard position, with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

1. $\sin \theta =$ ________ **2.** $\dfrac{r}{y} =$ ________

3. $\tan \theta =$ ________ **4.** $\sec \theta =$ ________

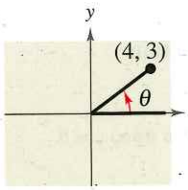
5. $\dfrac{x}{r} =$ ________ **6.** $\dfrac{x}{y} =$ ________

7. The acute positive angle that is formed by the terminal side of the angle θ and the horizontal axis is called the ________ angle of θ and is denoted by θ'.

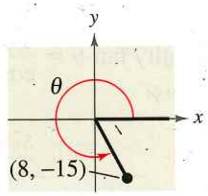
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–4, determine the exact values of the six trigonometric functions of the angle θ.

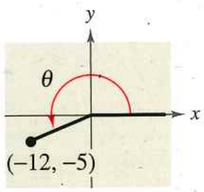
1. (a)



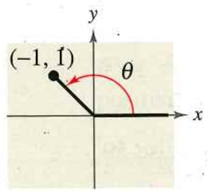
(b)



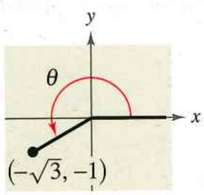
2. (a)



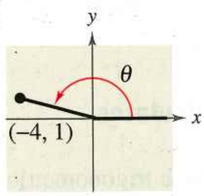
(b)



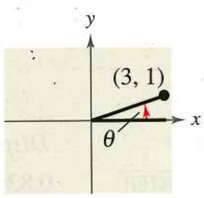
3. (a)



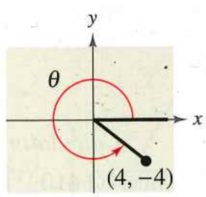
(b)



4. (a)



(b)



In Exercises 5–10, the point is on the terminal side of an angle in standard position. Determine the exact values of the six trigonometric functions of the angle.

5. $(7, 24)$ **6.** $(8, 15)$

7. $(-4, 10)$ **8.** $(-5, -2)$

9. $(-3.5, 6.8)$ **10.** $\left(3\frac{1}{2}, -7\frac{3}{4}\right)$

In Exercises 11–14, state the quadrant in which θ lies.

11. $\sin \theta < 0$ and $\cos \theta < 0$

12. $\sin \theta > 0$ and $\cos \theta > 0$

13. $\sin \theta > 0$ and $\tan \theta < 0$

14. $\sec \theta > 0$ and $\cot \theta < 0$

In Exercises 15–24, find the values of the six trigonometric functions of θ with the given constraint.

	Function Value	*Constraint*
15.	$\sin \theta = \frac{3}{5}$	θ lies in Quadrant II.
16.	$\cos \theta = -\frac{4}{5}$	θ lies in Quadrant III.
17.	$\tan \theta = -\frac{15}{8}$	$\sin \theta < 0$
18.	$\cos \theta = \frac{8}{17}$	$\tan \theta < 0$
19.	$\cot \theta = -3$	$\cos \theta > 0$
20.	$\csc \theta = 4$	$\cot \theta < 0$
21.	$\sec \theta = -2$	$\sin \theta > 0$
22.	$\sin \theta = 0$	$\sec \theta = -1$
23.	$\cot \theta$ is undefined.	$\pi/2 \leq \theta \leq 3\pi/2$
24.	$\tan \theta$ is undefined.	$\pi \leq \theta \leq 2\pi$

In Exercises 25–28, the terminal side of θ lies on the given line in the specified quadrant. Find the values of the six trigonometric functions of θ by finding a point on the line.

	Line	*Quadrant*
25.	$y = -x$	II
26.	$y = \frac{1}{3}x$	III
27.	$2x - y = 0$	III
28.	$4x + 3y = 0$	IV

In Exercises 29–36, evaluate the trigonometric function of the quadrant angle.

29. $\sin \pi$
30. $\csc \frac{3\pi}{2}$
31. $\sec \frac{3\pi}{2}$
32. $\sec \pi$
33. $\sin \frac{\pi}{2}$
34. $\cot \pi$
35. $\csc \pi$
36. $\cot \frac{\pi}{2}$

In Exercises 37–44, find the reference angle θ', and sketch θ and θ' in standard position.

37. $\theta = 203°$
38. $\theta = 309°$
39. $\theta = -245°$
40. $\theta = -145°$
41. $\theta = \frac{2\pi}{3}$
42. $\theta = \frac{7\pi}{4}$
43. $\theta = 3.5$
44. $\theta = \frac{11\pi}{3}$

In Exercises 45–58, evaluate the sine, cosine, and tangent of the angle without using a calculator.

45. $225°$
46. $300°$
47. $750°$
48. $-405°$
49. $-150°$
50. $-840°$
51. $\frac{4\pi}{3}$
52. $\frac{\pi}{4}$
53. $-\frac{\pi}{6}$
54. $-\frac{\pi}{2}$
55. $\frac{11\pi}{4}$
56. $\frac{10\pi}{3}$
57. $-\frac{3\pi}{2}$
58. $-\frac{25\pi}{4}$

In Exercises 59–64, find the indicated trigonometric value in the specified quadrant.

Function	*Quadrant*	*Trigonometric Value*
59. $\sin \theta = -\frac{3}{5}$	IV	$\cos \theta$
60. $\cot \theta = -3$	II	$\sin \theta$
61. $\tan \theta = \frac{3}{2}$	III	$\sec \theta$
62. $\csc \theta = -2$	IV	$\cot \theta$
63. $\cos \theta = \frac{5}{8}$	I	$\sec \theta$
64. $\sec \theta = -\frac{9}{4}$	III	$\tan \theta$

In Exercises 65–80, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is set in the correct angle mode.)

65. $\sin 10°$
66. $\sec 225°$
67. $\cos(-110°)$
68. $\csc(-330°)$
69. $\tan 304°$
70. $\cot 178°$
71. $\sec 72°$
72. $\tan(-188°)$
73. $\tan 4.5$
74. $\cot 1.35$
75. $\tan \frac{\pi}{9}$
76. $\tan\left(-\frac{\pi}{9}\right)$
77. $\sin(-0.65)$
78. $\sec 0.29$
79. $\cot\left(-\frac{11\pi}{8}\right)$
80. $\csc\left(-\frac{15\pi}{14}\right)$

In Exercises 81–86, find two solutions of the equation. Give your answers in degrees ($0° \le \theta < 360°$) and in radians ($0 \le \theta < 2\pi$). Do not use a calculator.

81. (a) $\sin \theta = \frac{1}{2}$ (b) $\sin \theta = -\frac{1}{2}$
82. (a) $\cos \theta = \frac{\sqrt{2}}{2}$ (b) $\cos \theta = -\frac{\sqrt{2}}{2}$
83. (a) $\csc \theta = \frac{2\sqrt{3}}{3}$ (b) $\cot \theta = -1$
84. (a) $\sec \theta = 2$ (b) $\sec \theta = -2$
85. (a) $\tan \theta = 1$ (b) $\cot \theta = -\sqrt{3}$
86. (a) $\sin \theta = \frac{\sqrt{3}}{2}$ (b) $\sin \theta = -\frac{\sqrt{3}}{2}$

Model It

87. ***Data Analysis: Meteorology*** The table shows the monthly normal temperatures (in degrees Fahrenheit) for selected months for New York City (N) and Fairbanks, Alaska (F). (Source: National Climatic Data Center)

Month	New York City, N	Fairbanks, F
January	33	−10
April	52	32
July	77	62
October	58	24
December	38	−6

(a) Use the *regression* feature of a graphing utility to find a model of the form

$$y = a\sin(bt + c) + d$$

for each city. Let t represent the month, with $t = 1$ corresponding to January.

Model It (continued)

(b) Use the models from part (a) to find the monthly normal temperatures for the two cities in February, March, May, June, August, September, and November.

(c) Compare the models for the two cities.

88. ***Sales*** A company that produces snowboards, which are seasonal products, forecasts monthly sales over the next 2 years to be

$$S = 23.1 + 0.442t + 4.3 \cos \frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ representing January 2006. Predict sales for each of the following months.

(a) February 2006

(b) February 2007

(c) June 2006

(d) June 2007

89. ***Harmonic Motion*** The displacement from equilibrium of an oscillating weight suspended by a spring is given by

$$y(t) = 2 \cos 6t$$

where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

90. ***Harmonic Motion*** The displacement from equilibrium of an oscillating weight suspended by a spring and subject to the damping effect of friction is given by

$$y(t) = 2e^{-t} \cos 6t$$

where y is the displacement (in centimeters) and t is the time (in seconds). Find the displacement when (a) $t = 0$, (b) $t = \frac{1}{4}$, and (c) $t = \frac{1}{2}$.

91. ***Electric Circuits*** The current I (in amperes) when 100 volts is applied to a circuit is given by

$$I = 5e^{-2t} \sin t$$

where t is the time (in seconds) after the voltage is applied. Approximate the current at $t = 0.7$ second after the voltage is applied.

92. ***Distance*** An airplane, flying at an altitude of 6 miles, is on a flight path that passes directly over an observer (see figure). If θ is the angle of elevation from the observer to the plane, find the distance d from the observer to the plane when (a) $\theta = 30°$, (b) $\theta = 90°$, and (c) $\theta = 120°$.

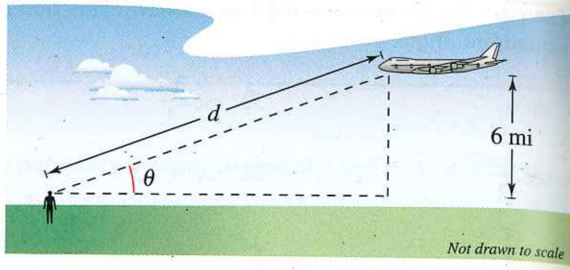


FIGURE FOR 92

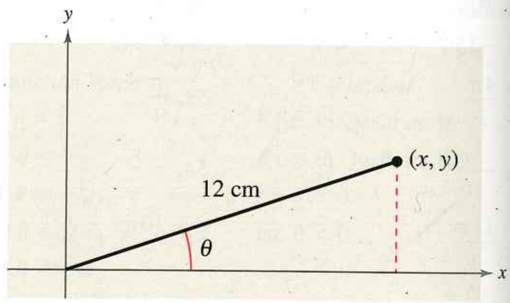
Synthesis

True or False? **In Exercises 93 and 94, determine whether the statement is true or false. Justify your answer.**

93. In each of the four quadrants, the signs of the secant function and sine function will be the same.

94. To find the reference angle for an angle θ (given in degrees), find the integer n such that $0 \le 360°n - \theta \le 360°$. The difference $360°n - \theta$ is the reference angle.

95. ***Writing*** Consider an angle in standard position with $r = 12$ centimeters, as shown in the figure. Write a short paragraph describing the changes in the values of x, y, $\sin \theta$, $\cos \theta$, and $\tan \theta$ as θ increases continuously from 0° to 90°.



96. ***Writing*** Explain how reference angles are used to find the trigonometric functions of obtuse angles.

Skills Review

In Exercises 97–106, graph the function. Identify the domain and any intercepts and asymptotes of the function.

97. $y = x^2 + 3x - 4$

98. $y = 2x^2 - 5x$

99. $f(x) = x^3 + 8$

100. $g(x) = x^4 + 2x^2 - 3$

101. $f(x) = \dfrac{x - 7}{x^2 + 4x + 4}$

102. $h(x) = \dfrac{x^2 - 1}{x + 5}$

103. $y = 2^{x-1}$

104. $y = 3^{x+1} + 2$

105. $y = \ln x^4$

106. $y = \log_{10}(x + 2)$

4.5 Exercises

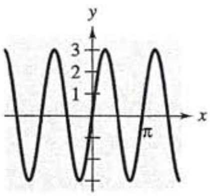
VOCABULARY CHECK: Fill in the blanks.

1. One period of a sine or cosine function function is called one ________ of the sine curve or cosine curve.
2. The ________ of a sine or cosine curve represents half the distance between the maximum and minimum values of the function.
3. The period of a sine or cosine function is given by ________.
4. For the function given by $y = a\sin(bx - c)$, $\dfrac{c}{b}$ represents the ________ ________ of the graph of the function.
5. For the function given by $y = d + a\cos(bx - c)$, d represents a ________ ________ of the graph of the function.

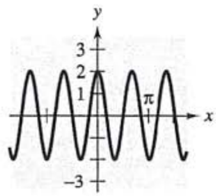
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–14, find the period and amplitude.

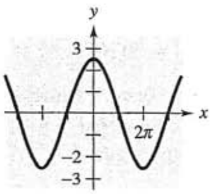
1. $y = 3\sin 2x$



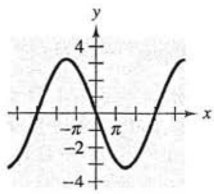
2. $y = 2\cos 3x$



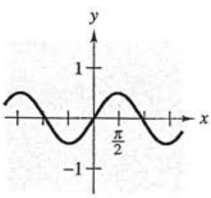
3. $y = \dfrac{5}{2}\cos\dfrac{x}{2}$



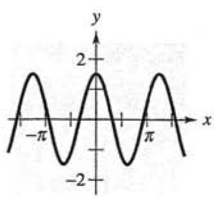
4. $y = -3\sin\dfrac{x}{3}$



5. $y = \dfrac{1}{2}\sin\dfrac{\pi x}{3}$



6. $y = \dfrac{3}{2}\cos\dfrac{\pi x}{2}$



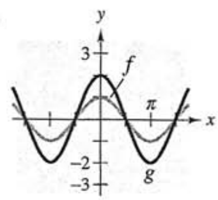
7. $y = -2\sin x$
8. $y = -\cos\dfrac{2x}{3}$
9. $y = 3\sin 10x$
10. $y = \frac{1}{3}\sin 8x$
11. $y = \dfrac{1}{2}\cos\dfrac{2x}{3}$
12. $y = \dfrac{5}{2}\cos\dfrac{x}{4}$
13. $y = \dfrac{1}{4}\sin 2\pi x$
14. $y = \dfrac{2}{3}\cos\dfrac{\pi x}{10}$

In Exercises 15–22, describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

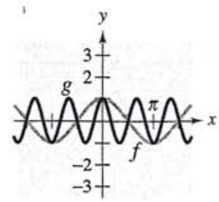
15. $f(x) = \sin x$
    $g(x) = \sin(x - \pi)$
16. $f(x) = \cos x$
    $g(x) = \cos(x + \pi)$
17. $f(x) = \cos 2x$
    $g(x) = -\cos 2x$
18. $f(x) = \sin 3x$
    $g(x) = \sin(-3x)$
19. $f(x) = \cos x$
    $g(x) = \cos 2x$
20. $f(x) = \sin x$
    $g(x) = \sin 3x$
21. $f(x) = \sin 2x$
    $g(x) = 3 + \sin 2x$
22. $f(x) = \cos 4x$
    $g(x) = -2 + \cos 4x$

In Exercises 23–26, describe the relationship between the graphs of *f* and *g*. Consider amplitude, period, and shifts.

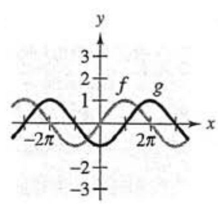
23.



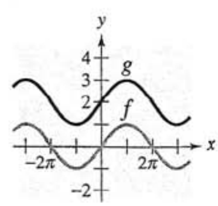
24.



25.



26.



In Exercises 27–34, graph f and g on the same set of coordinate axes. (Include two full periods.)

27. $f(x) = -2 \sin x$
$g(x) = 4 \sin x$

28. $f(x) = \sin x$
$g(x) = \sin \dfrac{x}{3}$

29. $f(x) = \cos x$
$g(x) = 1 + \cos x$

30. $f(x) = 2 \cos 2x$
$g(x) = -\cos 4x$

31. $f(x) = -\dfrac{1}{2} \sin \dfrac{x}{2}$
$g(x) = 3 - \dfrac{1}{2} \sin \dfrac{x}{2}$

32. $f(x) = 4 \sin \pi x$
$g(x) = 4 \sin \pi x - 3$

33. $f(x) = 2 \cos x$
$g(x) = 2 \cos(x + \pi)$

34. $f(x) = -\cos x$
$g(x) = -\cos(x - \pi)$

In Exercises 35–56, sketch the graph of the function. (Include two full periods.)

35. $y = 3 \sin x$

36. $y = \frac{1}{4} \sin x$

37. $y = \frac{1}{3} \cos x$

38. $y = 4 \cos x$

39. $y = \cos \dfrac{x}{2}$

40. $y = \sin 4x$

41. $y = \cos 2\pi x$

42. $y = \sin \dfrac{\pi x}{4}$

43. $y = -\sin \dfrac{2\pi x}{3}$

44. $y = -10 \cos \dfrac{\pi x}{6}$

45. $y = \sin\left(x - \dfrac{\pi}{4}\right)$

46. $y = \sin(x - \pi)$

47. $y = 3 \cos(x + \pi)$

48. $y = 4 \cos\left(x + \dfrac{\pi}{4}\right)$

49. $y = 2 - \sin \dfrac{2\pi x}{3}$

50. $y = -3 + 5 \cos \dfrac{\pi t}{12}$

51. $y = 2 + \frac{1}{10} \cos 60\pi x$

52. $y = 2 \cos x - 3$

53. $y = 3 \cos(x + \pi) - 3$

54. $y = 4 \cos\left(x + \dfrac{\pi}{4}\right) + 4$

55. $y = \dfrac{2}{3} \cos\left(\dfrac{x}{2} - \dfrac{\pi}{4}\right)$

56. $y = -3 \cos(6x + \pi)$

In Exercises 57–62, use a graphing utility to graph the function. Include two full periods. Be sure to choose an appropriate viewing window.

57. $y = -2 \sin(4x + \pi)$

58. $y = -4 \sin\left(\dfrac{2}{3}x - \dfrac{\pi}{3}\right)$

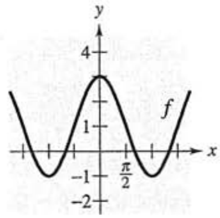
59. $y = \cos\left(2\pi x - \dfrac{\pi}{2}\right) + 1$

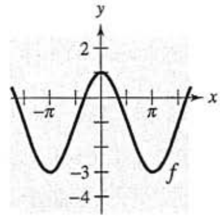
60. $y = 3 \cos\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right) - 2$

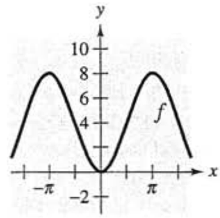
61. $y = -0.1 \sin\left(\dfrac{\pi x}{10} + \pi\right)$

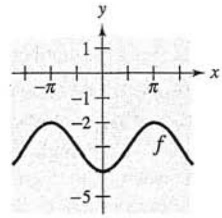
62. $y = \frac{1}{100} \sin 120\pi t$

Graphical Reasoning **In Exercises 63–66, find a and d for the function $f(x) = a \cos x + d$ such that the graph of f matches the figure.**

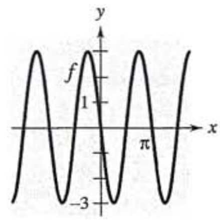
63.


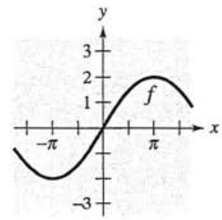
64.


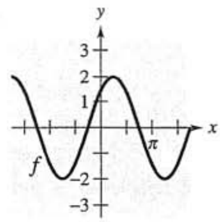
65.


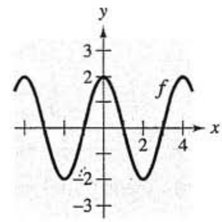
66.


Graphical Reasoning **In Exercises 67–70, find a, b, and c for the function $f(x) = a \sin(bx - c)$ such that the graph of f matches the figure.**

67.


68.


69.


70.


In Exercises 71 and 72, use a graphing utility to graph y_1 and y_2 in the interval $[-2\pi, 2\pi]$. Use the graphs to find real numbers x such that $y_1 = y_2$.

71. $y_1 = \sin x$
$y_2 = -\frac{1}{2}$

72. $y_1 = \cos x$
$y_2 = -1$

4.6 Exercises

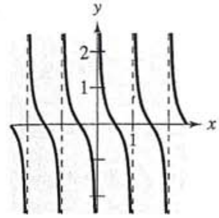
VOCABULARY CHECK: Fill in the blanks.

1. The graphs of the tangent, cotangent, secant, and cosecant functions all have ________ asymptotes.
2. To sketch the graph of a secant or cosecant function, first make a sketch of its corresponding ________ function.
3. For the functions given by $f(x) = g(x) \cdot \sin x$, $g(x)$ is called the ________ factor of the function $f(x)$.
4. The period of $y = \tan x$ is ________.
5. The domain of $y = \cot x$ is all real numbers such that ________.
6. The range of $y = \sec x$ is ________.
7. The period of $y = \csc x$ is ________.

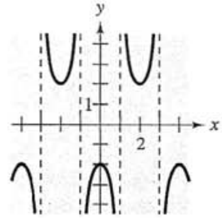
PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–6, match the function with its graph. State the period of the function. [The graphs are labeled (a), (b), (c), (d), (e), and (f).]

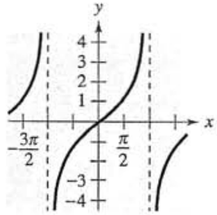
(a)



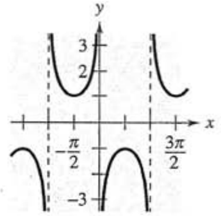
(b)



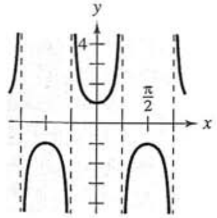
(c)



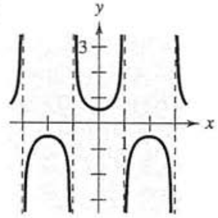
(d)



(e)



(f)



1. $y = \sec 2x$
2. $y = \tan \dfrac{x}{2}$
3. $y = \dfrac{1}{2} \cot \pi x$
4. $y = -\csc x$
5. $y = \dfrac{1}{2} \sec \dfrac{\pi x}{2}$
6. $y = -2 \sec \dfrac{\pi x}{2}$

In Exercises 7–30, sketch the graph of the function. Include two full periods.

7. $y = \frac{1}{3} \tan x$
8. $y = \frac{1}{4} \tan x$
9. $y = \tan 3x$
10. $y = -3 \tan \pi x$
11. $y = -\frac{1}{2} \sec x$
12. $y = \frac{1}{4} \sec x$
13. $y = \csc \pi x$
14. $y = 3 \csc 4x$
15. $y = \sec \pi x - 1$
16. $y = -2 \sec 4x + 2$
17. $y = \csc \dfrac{x}{2}$
18. $y = \csc \dfrac{x}{3}$
19. $y = \cot \dfrac{x}{2}$
20. $y = 3 \cot \dfrac{\pi x}{2}$
21. $y = \frac{1}{2} \sec 2x$
22. $y = -\frac{1}{2} \tan x$
23. $y = \tan \dfrac{\pi x}{4}$
24. $y = \tan(x + \pi)$
25. $y = \csc(\pi - x)$
26. $y = \csc(2x - \pi)$
27. $y = 2 \sec(x + \pi)$
28. $y = -\sec \pi x + 1$
29. $y = \dfrac{1}{4} \csc\left(x + \dfrac{\pi}{4}\right)$
30. $y = 2 \cot\left(x + \dfrac{\pi}{2}\right)$

In Exercises 31–40, use a graphing utility to graph the function. Include two full periods.

31. $y = \tan \dfrac{x}{3}$
32. $y = -\tan 2x$
33. $y = -2 \sec 4x$
34. $y = \sec \pi x$
35. $y = \tan\left(x - \dfrac{\pi}{4}\right)$
36. $y = \dfrac{1}{4} \cot\left(x - \dfrac{\pi}{2}\right)$
37. $y = -\csc(4x - \pi)$
38. $y = 2 \sec(2x - \pi)$
39. $y = 0.1 \tan\left(\dfrac{\pi x}{4} + \dfrac{\pi}{4}\right)$
40. $y = \dfrac{1}{3} \sec\left(\dfrac{\pi x}{2} + \dfrac{\pi}{2}\right)$

98. *Data Analysis: Meteorology* The times S of sunset (Greenwich Mean Time) at 40° north latitude on the 15th of each month are: 1(16:59), 2(17:35), 3(18:06), 4(18:38), 5(19:08), 6(19:30), 7(19:28), 8(18:57), 9(18:09), 10(17:21), 11(16:44), 12(16:36). The month is represented by t, with $t = 1$ corresponding to January. A model (in which minutes have been converted to the decimal parts of an hour) for the data is

$$S(t) = 18.09 + 1.41 \sin\left(\frac{\pi t}{6} + 4.60\right).$$

(a) Use a graphing utility to graph the data points and the model in the same viewing window.

(b) What is the period of the model? Is it what you expected? Explain.

(c) What is the amplitude of the model? What does it represent in the model? Explain.

4.6 **In Exercises 99–106, sketch a graph of the function. Include two full periods.**

99. $f(x) = \tan x$

100. $f(t) = \tan\left(t - \frac{\pi}{4}\right)$

101. $f(x) = \cot x$

102. $g(t) = 2 \cot 2t$

103. $f(x) = \sec x$

104. $h(t) = \sec\left(t - \frac{\pi}{4}\right)$

105. $f(x) = \csc x$

106. $f(t) = 3 \csc\left(2t + \frac{\pi}{4}\right)$

In Exercises 107 and 108, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.

107. $f(x) = x \cos x$

108. $g(x) = x^4 \cos x$

4.7 **In Exercises 109–114, evaluate the expression. If necessary, round your answer to two decimal places.**

109. $\arcsin\left(-\frac{1}{2}\right)$

110. $\arcsin(-1)$

111. $\arcsin 0.4$

112. $\arcsin 0.213$

113. $\sin^{-1}(-0.44)$

114. $\sin^{-1} 0.89$

In Exercises 115–118, evaluate the expression without the aid of a calculator.

115. $\arccos \frac{\sqrt{3}}{2}$

116. $\arccos \frac{\sqrt{2}}{2}$

117. $\cos^{-1}(-1)$

118. $\cos^{-1} \frac{\sqrt{3}}{2}$

In Exercises 119–122, use a calculator to evaluate the expression. Round your answer to two decimal places.

119. $\arccos 0.324$

120. $\arccos(-0.888)$

121. $\tan^{-1}(-1.5)$

122. $\tan^{-1} 8.2$

In Exercises 123–126, use a graphing utility to graph the function.

123. $f(x) = 2 \arcsin x$

124. $f(x) = 3 \arccos x$

125. $f(x) = \arctan \frac{x}{2}$

126. $f(x) = -\arcsin 2x$

In Exercises 127–130, find the exact value of the expression.

127. $\cos\left(\arctan \frac{3}{4}\right)$

128. $\tan\left(\arccos \frac{3}{5}\right)$

129. $\sec\left(\arctan \frac{12}{5}\right)$

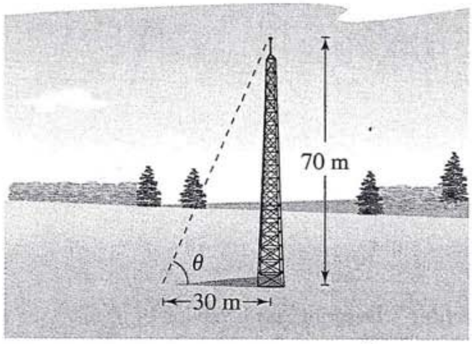
130. $\cot\left[\arcsin\left(-\frac{12}{13}\right)\right]$

In Exercises 131 and 132, write an algebraic expression that is equivalent to the expression.

131. $\tan\left(\arccos \frac{x}{2}\right)$

132. $\sec[\arcsin(x - 1)]$

4.8 **133.** *Angle of Elevation* The height of a radio transmission tower is 70 meters, and it casts a shadow of length 30 meters (see figure). Find the angle of elevation of the sun.



134. *Height* Your football has landed at the edge of the roof of your school building. When you are 25 feet from the base of the building, the angle of elevation to your football is 21°. How high off the ground is your football?

135. *Distance* From city A to city B, a plane flies 650 miles at a bearing of 48°. From city B to city C, the plane flies 810 miles at a bearing of 115°. Find the distance from city A to city C and the bearing from city A to city C.

4.8 Exercises

VOCABULARY CHECK: Fill in the blanks.

1. An angle that measures from the horizontal upward to an object is called the angle of ________, whereas an angle that measures from the horizontal downward to an object is called the angle of ________.
2. A ________ measures the acute angle a path or line of sight makes with a fixed north-south line.
3. A point that moves on a coordinate line is said to be in simple ________ ________ if its distance d from the origin at time t is given by either $d = a \sin \omega t$ or $d = a \cos \omega t$.

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–10, solve the right triangle shown in the figure. Round your answers to two decimal places.

1. $A = 20°, \quad b = 10$
2. $B = 54°, \quad c = 15$
3. $B = 71°, \quad b = 24$
4. $A = 8.4°, \quad a = 40.5$
5. $a = 6, \quad b = 10$
6. $a = 25, \quad c = 35$
7. $b = 16, \quad c = 52$
8. $b = 1.32, \quad c = 9.45$
9. $A = 12°15', \quad c = 430.5$
10. $B = 65°12', \quad a = 14.2$

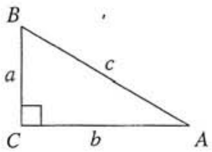


FIGURE FOR 1–10

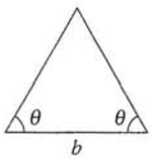
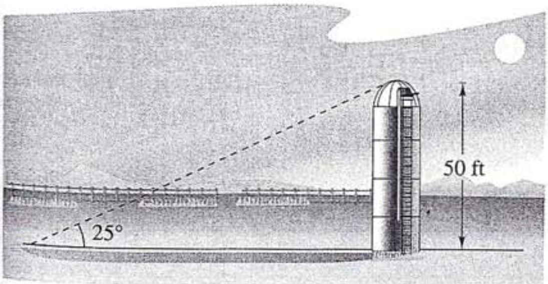


FIGURE FOR 11–14

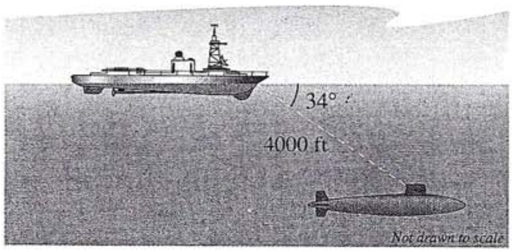
In Exercises 11–14, find the altitude of the isosceles triangle shown in the figure. Round your answers to two decimal places.

11. $\theta = 52°, \quad b = 4$ inches
12. $\theta = 18°, \quad b = 10$ meters
13. $\theta = 41°, \quad b = 46$ inches
14. $\theta = 27°, \quad b = 11$ feet

15. ***Length*** The sun is 25° above the horizon. Find the length of a shadow cast by a silo that is 50 feet tall (see figure).



16. ***Length*** The sun is 20° above the horizon. Find the length of a shadow cast by a building that is 600 feet tall.
17. ***Height*** A ladder 20 feet long leans against the side of a house. Find the height from the top of the ladder to the ground if the angle of elevation of the ladder is 80°.
18. ***Height*** The length of a shadow of a tree is 125 feet when the angle of elevation of the sun is 33°. Approximate the height of the tree.
19. ***Height*** From a point 50 feet in front of a church, the angles of elevation to the base of the steeple and the top of the steeple are 35° and 47° 40′, respectively.
    (a) Draw right triangles that give a visual representation of the problem. Label the known and unknown quantities.
    (b) Use a trigonometric function to write an equation involving the unknown quantity.
    (c) Find the height of the steeple.
20. ***Height*** You are standing 100 feet from the base of a platform from which people are bungee jumping. The angle of elevation from your position to the top of the platform from which they jump is 51°. From what height are the people jumping?
21. ***Depth*** The sonar of a navy cruiser detects a submarine that is 4000 feet from the cruiser. The angle between the water line and the submarine is 34° (see figure). How deep is the submarine?



22. ***Angle of Elevation*** An engineer erects a 75-foot cellular telephone tower. Find the angle of elevation to the top of the tower at a point on level ground 50 feet from its base.

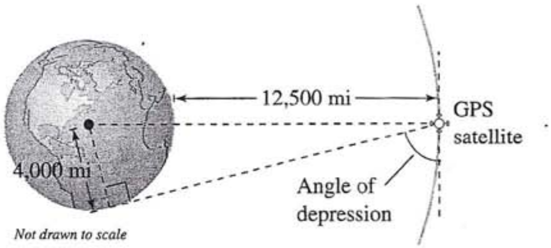
23. ***Angle of Elevation*** The height of an outdoor basketball backboard is $12\frac{1}{2}$ feet, and the backboard casts a shadow $17\frac{1}{3}$ feet long.

(a) Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities.

(b) Use a trigonometric function to write an equation involving the unknown quantity.

(c) Find the angle of elevation of the sun.

24. ***Angle of Depression*** A Global Positioning System satellite orbits 12,500 miles above Earth's surface (see figure). Find the angle of depression from the satellite to the horizon. Assume the radius of Earth is 4000 miles.



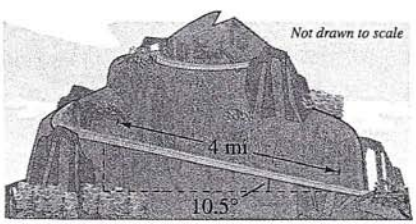
25. ***Angle of Depression*** A cellular telephone tower that is 150 feet tall is placed on top of a mountain that is 1200 feet above sea level. What is the angle of depression from the top of the tower to a cell phone user who is 5 horizontal miles away and 400 feet above sea level?

26. ***Airplane Ascent*** During takeoff, an airplane's angle of ascent is 18° and its speed is 275 feet per second.

(a) Find the plane's altitude after 1 minute.

(b) How long will it take the plane to climb to an altitude of 10,000 feet?

27. ***Mountain Descent*** A sign on a roadway at the top of a mountain indicates that for the next 4 miles the grade is 10.5° (see figure). Find the change in elevation over that distance for a car descending the mountain.



28. ***Mountain Descent*** A roadway sign at the top of a mountain indicates that for the next 4 miles the grade is 12%. Find the angle of the grade and the change in elevation over the 4 miles for a car descending the mountain.

29. ***Navigation*** An airplane flying at 600 miles per hour has a bearing of 52°. After flying for 1.5 hours, how far north and how far east will the plane have traveled from its point of departure?

30. ***Navigation*** A jet leaves Reno, Nevada and is headed toward Miami, Florida at a bearing of 100°. The distance between the two cities is approximately 2472 miles.

(a) How far north and how far west is Reno relative to Miami?

(b) If the jet is to return directly to Reno from Miami, at what bearing should it travel?

31. ***Navigation*** A ship leaves port at noon and has a bearing of S 29° W. The ship sails at 20 knots.

(a) How many nautical miles south and how many nautical miles west will the ship have traveled by 6:00 P.M.?

(b) At 6:00 P.M., the ship changes course to due west. Find the ship's bearing and distance from the port of departure at 7:00 P.M.

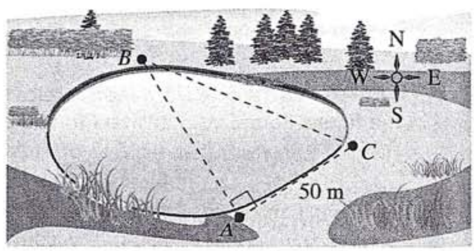
32. ***Navigation*** A privately owned yacht leaves a dock in Myrtle Beach, South Carolina and heads toward Freeport in the Bahamas at a bearing of S 1.4° E. The yacht averages a speed of 20 knots over the 428 nautical-mile trip.

(a) How long will it take the yacht to make the trip?

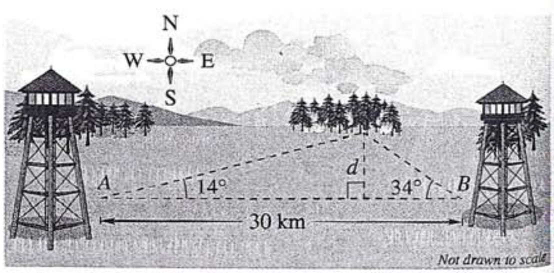
(b) How far east and south is the yacht after 12 hours?

(c) If a plane leaves Myrtle Beach to fly to Freeport, what bearing should be taken?

33. ***Surveying*** A surveyor wants to find the distance across a swamp (see figure). The bearing from A to B is N 32° W. The surveyor walks 50 meters from A, and at the point C the bearing to B is N 68° W. Find (a) the bearing from A to C and (b) the distance from A to B.



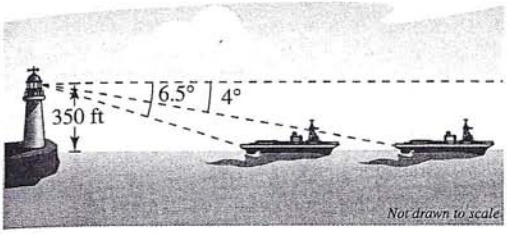
34. ***Location of a Fire*** Two fire towers are 30 kilometers apart, where tower A is due west of tower B. A fire is spotted from the towers, and the bearings from A and B are E 14° N and W 34° N, respectively (see figure). Find the distance d of the fire from the line segment AB.



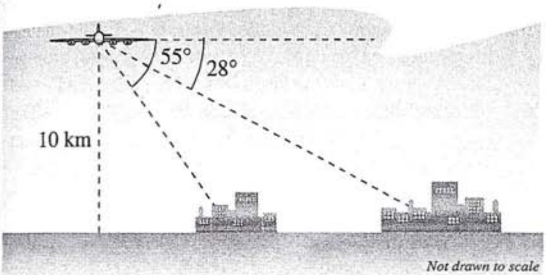
35. ***Navigation*** A ship is 45 miles east and 30 miles south of port. The captain wants to sail directly to port. What bearing should be taken?

36. ***Navigation*** An airplane is 160 miles north and 85 miles east of an airport. The pilot wants to fly directly to the airport. What bearing should be taken?

37. ***Distance*** An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4° and 6.5° (see figure). How far apart are the ships?



38. ***Distance*** A passenger in an airplane at an altitude of 10 kilometers sees two towns directly to the east of the plane. The angles of depression to the towns are 28° and 55° (see figure). How far apart are the towns?



39. ***Altitude*** A plane is observed approaching your home and you assume that its speed is 550 miles per hour. The angle of elevation of the plane is 16° at one time and 57° one minute later. Approximate the altitude of the plane.

40. ***Height*** While traveling across flat land, you notice a mountain directly in front of you. The angle of elevation to the peak is 2.5°. After you drive 17 miles closer to the mountain, the angle of elevation is 9°. Approximate the height of the mountain.

Geometry **In Exercises 41 and 42, find the angle α between two nonvertical lines L_1 and L_2. The angle α satisfies the equation**

$$\tan \alpha = \left| \frac{m_2 - m_1}{1 + m_2 m_1} \right|$$

where m_1 and m_2 are the slopes of L_1 and L_2, respectively. (Assume that $m_1 m_2 \neq -1$.)

41. L_1: $3x - 2y = 5$
L_2: $x + y = 1$

42. L_1: $2x - y = 8$
L_2: $x - 5y = -4$

43. ***Geometry*** Determine the angle between the diagonal of a cube and the diagonal of its base, as shown in the figure.

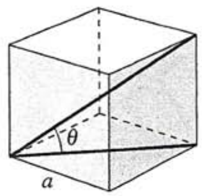


FIGURE FOR 43

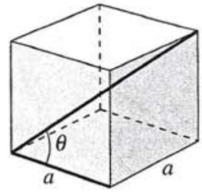


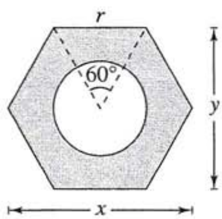
FIGURE FOR 44

44. ***Geometry*** Determine the angle between the diagonal of a cube and its edge, as shown in the figure.

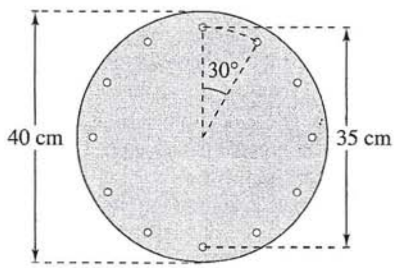
45. ***Geometry*** Find the length of the sides of a regular pentagon inscribed in a circle of radius 25 inches.

46. ***Geometry*** Find the length of the sides of a regular hexagon inscribed in a circle of radius 25 inches.

47. ***Hardware*** Write the distance y across the flat sides of a hexagonal nut as a function of r, as shown in the figure.



48. ***Bolt Holes*** The figure shows a circular piece of sheet metal that has a diameter of 40 centimeters and contains 12 equally spaced bolt holes. Determine the straight-line distance between the centers of consecutive bolt holes.



4.7 Exercises

VOCABULARY CHECK: Fill in the blanks.

Function	*Alternative Notation*	*Domain*	*Range*
1. $y = \arcsin x$	________	________	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
2. ________	$y = \cos^{-1} x$	$-1 \le x \le 1$	________
3. $y = \arctan x$	________	________	________

PREREQUISITE SKILLS REVIEW: Practice and review algebra skills needed for this section at **www.Eduspace.com.**

In Exercises 1–16, evaluate the expression without using a calculator.

1. $\arcsin \frac{1}{2}$
2. $\arcsin 0$
3. $\arccos \frac{1}{2}$
4. $\arccos 0$
5. $\arctan \frac{\sqrt{3}}{3}$
6. $\arctan(-1)$
7. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
8. $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
9. $\arctan(-\sqrt{3})$
10. $\arctan \sqrt{3}$
11. $\arccos\left(-\frac{1}{2}\right)$
12. $\arcsin \frac{\sqrt{2}}{2}$
13. $\sin^{-1} \frac{\sqrt{3}}{2}$
14. $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$
15. $\tan^{-1} 0$
16. $\cos^{-1} 1$

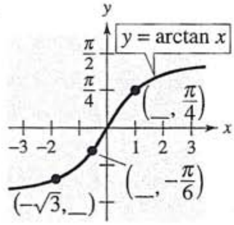
In Exercises 17 and 18, use a graphing utility to graph *f*, *g*, and *y* = *x* in the same viewing window to verify geometrically that *g* is the inverse function of *f*. (Be sure to restrict the domain of *f* properly.)

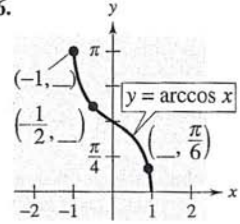
17. $f(x) = \sin x, \quad g(x) = \arcsin x$
18. $f(x) = \tan x, \quad g(x) = \arctan x$

In Exercises 19–34, use a calculator to evaluate the expression. Round your result to two decimal places.

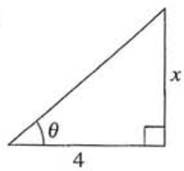
19. $\arccos 0.28$
20. $\arcsin 0.45$
21. $\arcsin(-0.75)$
22. $\arccos(-0.7)$
23. $\arctan(-3)$
24. $\arctan 15$
25. $\sin^{-1} 0.31$
26. $\cos^{-1} 0.26$
27. $\arccos(-0.41)$
28. $\arcsin(-0.125)$
29. $\arctan 0.92$
30. $\arctan 2.8$
31. $\arcsin \frac{3}{4}$
32. $\arccos\left(-\frac{1}{3}\right)$
33. $\tan^{-1} \frac{7}{2}$
34. $\tan^{-1}\left(-\frac{95}{7}\right)$

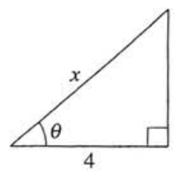
In Exercises 35 and 36, determine the missing coordinates of the points on the graph of the function.

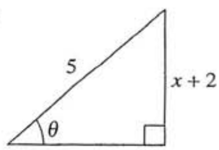
35. 

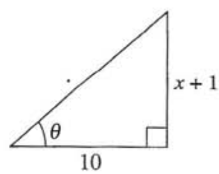
36. 

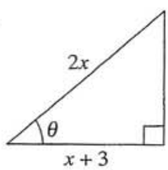
In Exercises 37–42, use an inverse trigonometric function to write θ as a function of x.

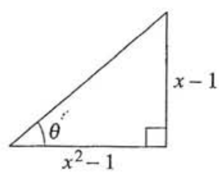
37. 

38. 

39. 

40. 

41. 

42. 

In Exercises 43–48, use the properties of inverse trigonometric functions to evaluate the expression.

43. $\sin(\arcsin 0.3)$
44. $\tan(\arctan 25)$
45. $\cos[\arccos(-0.1)]$
46. $\sin[\arcsin(-0.2)]$
47. $\arcsin(\sin 3\pi)$
48. $\arccos\left(\cos \frac{7\pi}{2}\right)$

In Exercises 49–58, find the exact value of the expression. (*Hint:* Sketch a right triangle.)

49. $\sin\left(\arctan \frac{3}{4}\right)$

50. $\sec\left(\arcsin \frac{4}{5}\right)$

51. $\cos(\tan^{-1} 2)$

52. $\sin\left(\cos^{-1} \frac{\sqrt{5}}{5}\right)$

53. $\cos\left(\arcsin \frac{5}{13}\right)$

54. $\csc\left[\arctan\left(-\frac{5}{12}\right)\right]$

55. $\sec\left[\arctan\left(-\frac{3}{5}\right)\right]$

56. $\tan\left[\arcsin\left(-\frac{3}{4}\right)\right]$

57. $\sin\left[\arccos\left(-\frac{2}{3}\right)\right]$

58. $\cot\left(\arctan \frac{5}{8}\right)$

In Exercises 59–68, write an algebraic expression that is equivalent to the expression. (*Hint:* Sketch a right triangle, as demonstrated in Example 7.)

59. $\cot(\arctan x)$

60. $\sin(\arctan x)$

61. $\cos(\arcsin 2x)$

62. $\sec(\arctan 3x)$

63. $\sin(\arccos x)$

64. $\sec[\arcsin(x - 1)]$

65. $\tan\left(\arccos \frac{x}{3}\right)$

66. $\cot\left(\arctan \frac{1}{x}\right)$

67. $\csc\left(\arctan \frac{x}{\sqrt{2}}\right)$

68. $\cos\left(\arcsin \frac{x - h}{r}\right)$

In Exercises 69 and 70, use a graphing utility to graph *f* and *g* in the same viewing window to verify that the two functions are equal. Explain why they are equal. Identify any asymptotes of the graphs.

69. $f(x) = \sin(\arctan 2x), \quad g(x) = \dfrac{2x}{\sqrt{1 + 4x^2}}$

70. $f(x) = \tan\left(\arccos \frac{x}{2}\right), \quad g(x) = \dfrac{\sqrt{4 - x^2}}{x}$

In Exercises 71–74, fill in the blank.

71. $\arctan \frac{9}{x} = \arcsin(\quad), \quad x \neq 0$

72. $\arcsin \frac{\sqrt{36 - x^2}}{6} = \arccos(\quad), \quad 0 \le x \le 6$

73. $\arccos \frac{3}{\sqrt{x^2 - 2x + 10}} = \arcsin(\quad)$

74. $\arccos \frac{x - 2}{2} = \arctan(\quad), \quad |x - 2| \le 2$

In Exercises 75 and 76, sketch a graph of the function and compare the graph of g with the graph of $f(x) = \arcsin x$.

75. $g(x) = \arcsin(x - 1)$

76. $g(x) = \arcsin \frac{x}{2}$

In Exercises 77–82, sketch a graph of the function.

77. $y = 2 \arccos x$

78. $g(t) = \arccos(t + 2)$

79. $f(x) = \arctan 2x$

80. $f(x) = \frac{\pi}{2} + \arctan x$

81. $h(v) = \tan(\arccos v)$

82. $f(x) = \arccos \frac{x}{4}$

In Exercises 83–88, use a graphing utility to graph the function.

83. $f(x) = 2 \arccos(2x)$

84. $f(x) = \pi \arcsin(4x)$

85. $f(x) = \arctan(2x - 3)$

86. $f(x) = -3 + \arctan(\pi x)$

87. $f(x) = \pi - \sin^{-1}\left(\frac{2}{3}\right)$

88. $f(x) = \frac{\pi}{2} + \cos^{-1}\left(\frac{1}{\pi}\right)$

In Exercises 89 and 90, write the function in terms of the sine function by using the identity

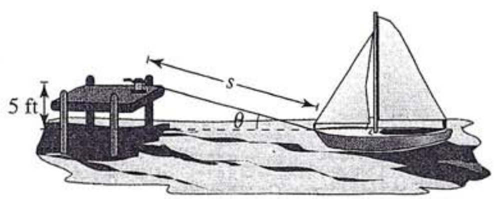
$$A \cos \omega t + B \sin \omega t = \sqrt{A^2 + B^2} \sin\left(\omega t + \arctan \frac{A}{B}\right).$$

Use a graphing utility to graph both forms of the function. What does the graph imply?

89. $f(t) = 3 \cos 2t + 3 \sin 2t$

90. $f(t) = 4 \cos \pi t + 3 \sin \pi t$

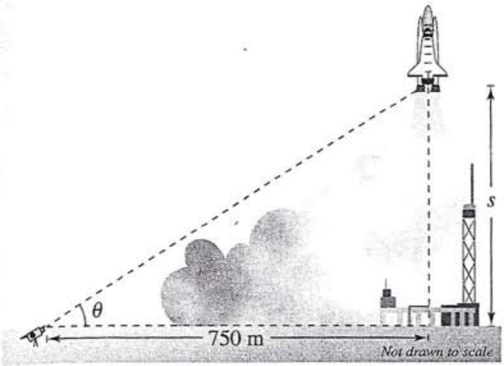
91. ***Docking a Boat*** A boat is pulled in by means of a winch located on a dock 5 feet above the deck of the boat (see figure). Let θ be the angle of elevation from the boat to the winch and let s be the length of the rope from the winch to the boat.



(a) Write θ as a function of s.

(b) Find θ when $s = 40$ feet and $s = 20$ feet.

92. ***Photography*** A television camera at ground level is filming the lift-off of a space shuttle at a point 750 meters from the launch pad (see figure). Let θ be the angle of elevation to the shuttle and let s be the height of the shuttle.



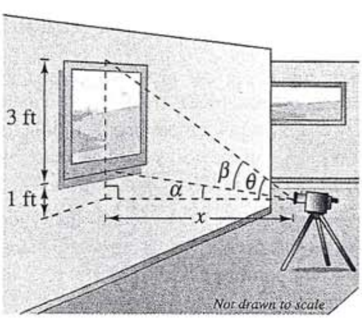
(a) Write θ as a function of s.

(b) Find θ when $s = 300$ meters and $s = 1200$ meters.

Model It

93. ***Photography*** A photographer is taking a picture of a three-foot-tall painting hung in an art gallery. The camera lens is 1 foot below the lower edge of the painting (see figure). The angle β subtended by the camera lens x feet from the painting is

$$\beta = \arctan \frac{3x}{x^2 + 4}, \quad x > 0.$$

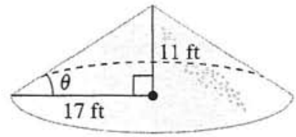


(a) Use a graphing utility to graph β as a function of x.

(b) Move the cursor along the graph to approximate the distance from the picture when β is maximum.

(c) Identify the asymptote of the graph and discuss its meaning in the context of the problem.

94. ***Granular Angle of Repose*** Different types of granular substances naturally settle at different angles when stored in cone-shaped piles. This angle θ is called the *angle of repose* (see figure). When rock salt is stored in a cone-shaped pile 11 feet high, the diameter of the pile's base is about 34 feet. (Source: Bulk-Store Structures, Inc.)



(a) Find the angle of repose for rock salt.

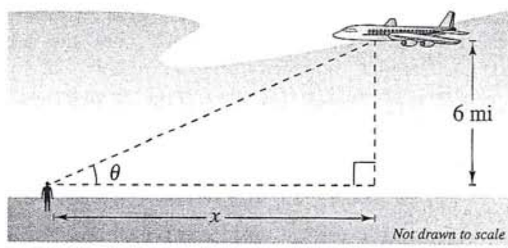
(b) How tall is a pile of rock salt that has a base diameter of 40 feet?

95. ***Granular Angle of Repose*** When whole corn is stored in a cone-shaped pile 20 feet high, the diameter of the pile's base is about 82 feet.

(a) Find the angle of repose for whole corn.

(b) How tall is a pile of corn that has a base diameter of 100 feet?

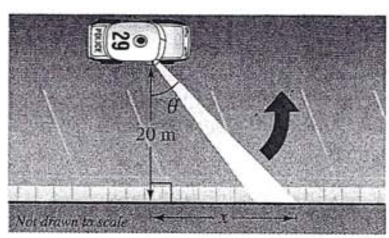
96. ***Angle of Elevation*** An airplane flies at an altitude of 6 miles toward a point directly over an observer. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 7$ miles and $x = 1$ mile.

97. ***Security Patrol*** A security car with its spotlight on is parked 20 meters from a warehouse. Consider θ and x as shown in the figure.



(a) Write θ as a function of x.

(b) Find θ when $x = 5$ meters and $x = 12$ meters.